KANTO GAKUIN UNIVERSITY

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Room1 Hazard and risk assessment e90064

Probabilistic tsunami inundation assessment using mode decomposition method −A case study for Kamakura city−

Yo Fukutani[1] ・Shuji Moriguchi[2] ・Kenjiro Terada[2] ・Yu Otake[2]

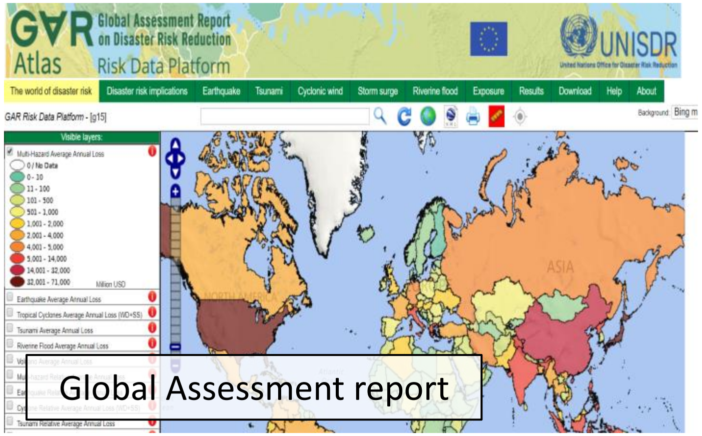
[1] Kanto Gakuin University [2]Tohoku University

Background and Objective

- Probabilistic Tsunami Hazard Assessment (PTHA) should be used for a basis of height of seawall, nuclear safety, and risk assessment in international organizations, insurance company or real estate sectors.
- In PTHA, we need to calculate the possible frequency distributions of tsunami hazards, which usually requires a significant computational load.


Seawall


Nuclear safety



Global Assessment report

Past studies of Probabilistic Tsunami Hazard Analysis

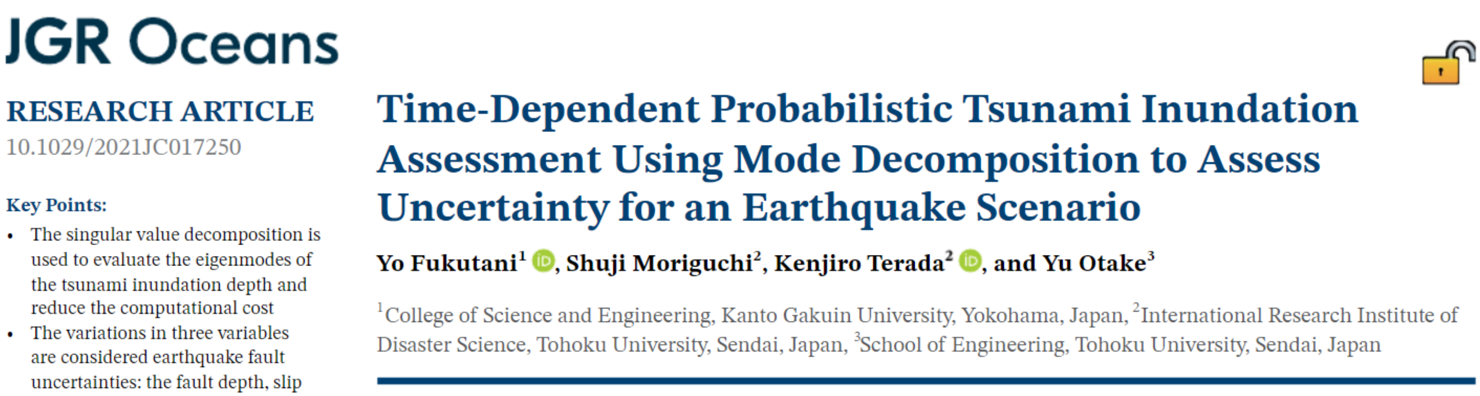
- Logic tree approach (Geist & Parsons (2006), Annaka et al. (2007) etc.)
- Stochastic rupture model (Goda et al. (2014), Miyashita et al. (2020) etc.)
- KL expansion method (LeVeque et al. (2016), Sepulveda et al. (2017) etc.)
- Green's function approach (Løvholt et al. (2015), Lorito et al. (2015) etc.)

➡ **Most of them require a large computational load because they assume hundreds to thousands of earthquake fault cases.**

Background and Objective

【Objective】

- Carry out the probabilistic tsunami inundation assessment considering the uncertainty of the earthquake fault **with reducing the computational load by evaluating the spatial correlation mode of tsunami inundation depth** using the tsunami numerical analysis and the eigen-orthogonal decomposition.



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RESEARCH ARTICLE

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Key Points:

- The singular value decomposition is used to evaluate the eigenmodes of the tsunami inundation depth and reduce the computational cost
- The variations in three variables are considered earthquake fault uncertainties: the fault depth, slip

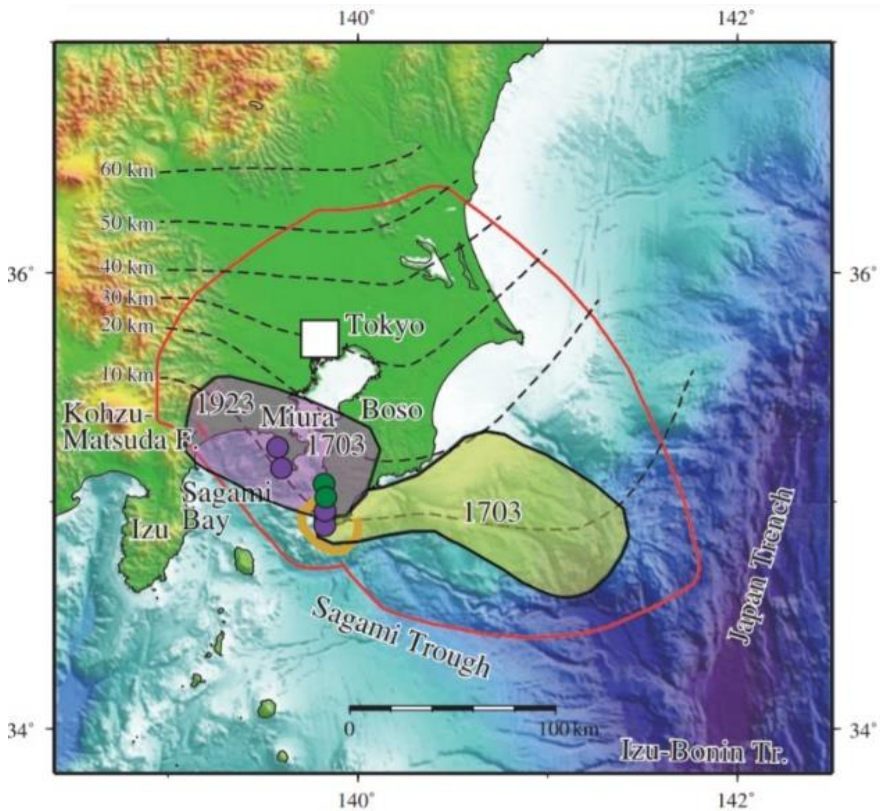
Time-Dependent Probabilistic Tsunami Inundation Assessment Using Mode Decomposition to Assess Uncertainty for an Earthquake Scenario

Yo Fukutani[1], Shuji Moriguchi[2], Kenjiro Terada[2], and Yu Otake[3]

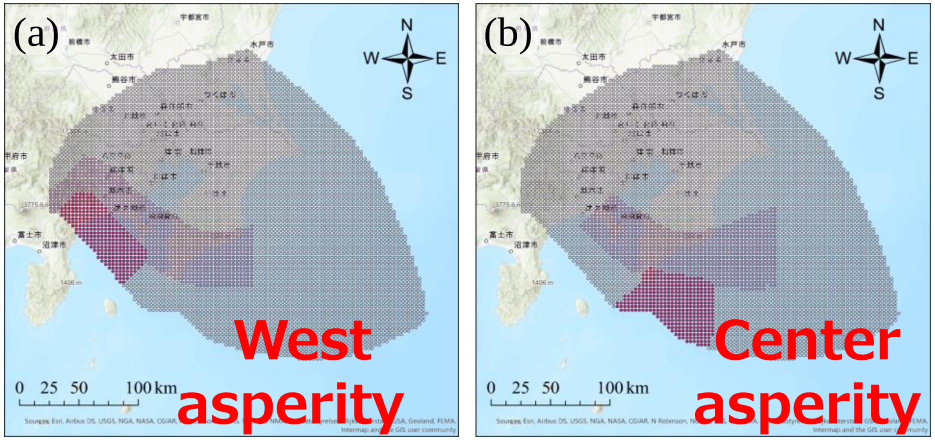
[1]College of Science and Engineering, Kanto Gakuin University, Yokohama, Japan, [2]International Research Institute of Disaster Science, Tohoku University, Sendai, Japan, [3]School of Engineering, Tohoku University, Sendai, Japan

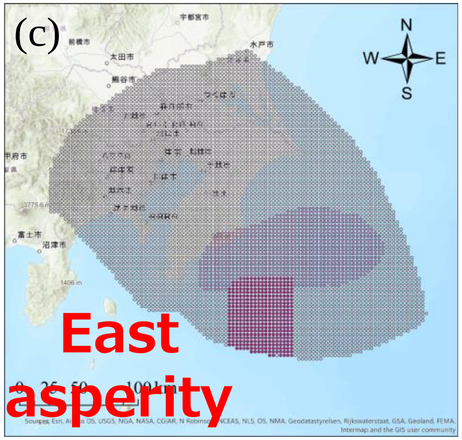
Tsunami source - Sagami trough earthquake -

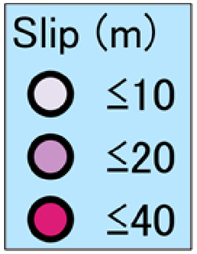
- Three patterns of the Sagami trough megathrust earthquake (Mw 8.7) published by the committee of the model for Tokyo Metropolitan Earthquake (2014).
- The earthquake has three levels of slip: a very large slip region (37.67 m), a large slip region (18.83 m), and a background region (9.42 m) to satisfy the Mw 8.7 for the entire fault.



The Sagami trough megathrust earthquake
(Satake, K. (2015))







Asperity location of the Sagami trough megathrust earthquake

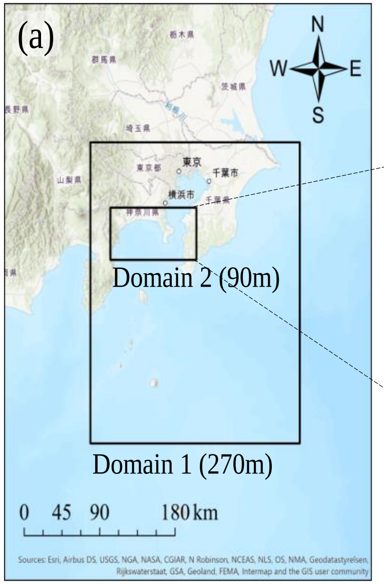
Tsunami numerical simulation

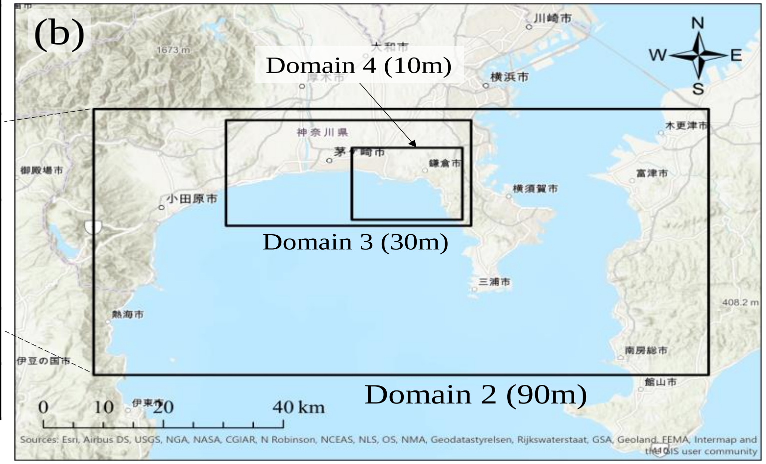
- We calculated the initial tsunami water level from the fault parameters using the equation of Okada (1985), and performed the tsunami numerical simulation for 3 hours after the earthquake occurs under the following conditions.

Tsunami numerical simulation condition

Governing equation (TUNAMI-N2)	2 -D nonlinear shallow water equations $\begin{cases} \frac{\partial \eta}{\partial t} + \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} = 0 \\ \frac{\partial M}{\partial t} + \frac{\partial}{\partial x}\left[\frac{M^2}{D}\right] + \frac{\partial}{\partial y}\left[\frac{MN}{D}\right] + gD\frac{\partial \eta}{\partial x} + \frac{gn^2}{D^{7/3}} M\sqrt{M^2+N^2} = 0 \\ \frac{\partial N}{\partial t} + \frac{\partial}{\partial x}\left[\frac{MN}{D}\right] + \frac{\partial}{\partial y}\left[\frac{N^2}{D}\right] + gD\frac{\partial \eta}{\partial y} + \frac{gn^2}{D^{7/3}} N\sqrt{M^2+N^2} = 0 \end{cases}$
Integration method	Staggered-leap frog method
Initial condition	Initial water level from the fault parameters using Okada(1985)
Boundary condition	Open boundary
Tide	T.P. +0.9 m (Kanagawa prefecture)
Mesh size	270 m → 90 m → 30 m → 10 m
Time interval	0.6 s

Simulation area (4 nested domains)

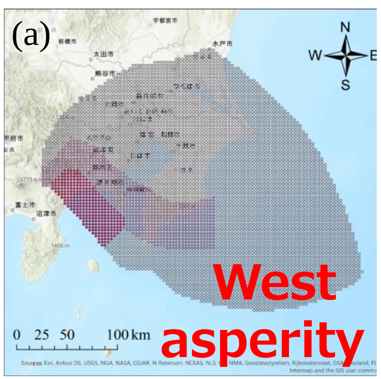


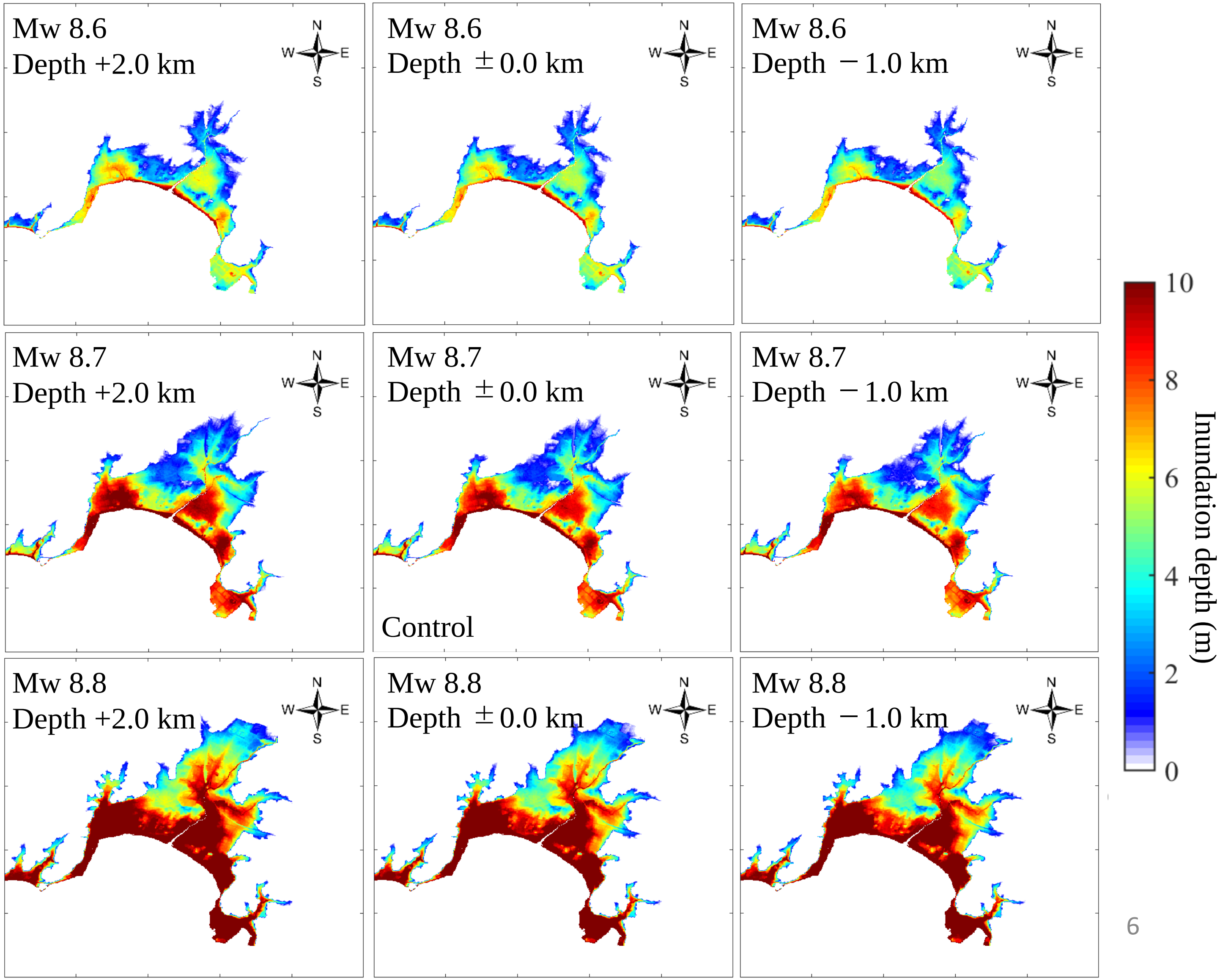


Uncertainty of earthquake fault

- Consideration of changes in fault slip (Mw8.7±0.1) and fault depth (+2km, ±0km, -1km) → Tsunami numerical simulations of 3 x 3 = 9 cases

West asperity model (Kamakura)



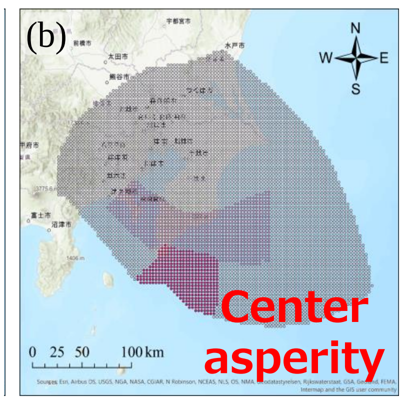


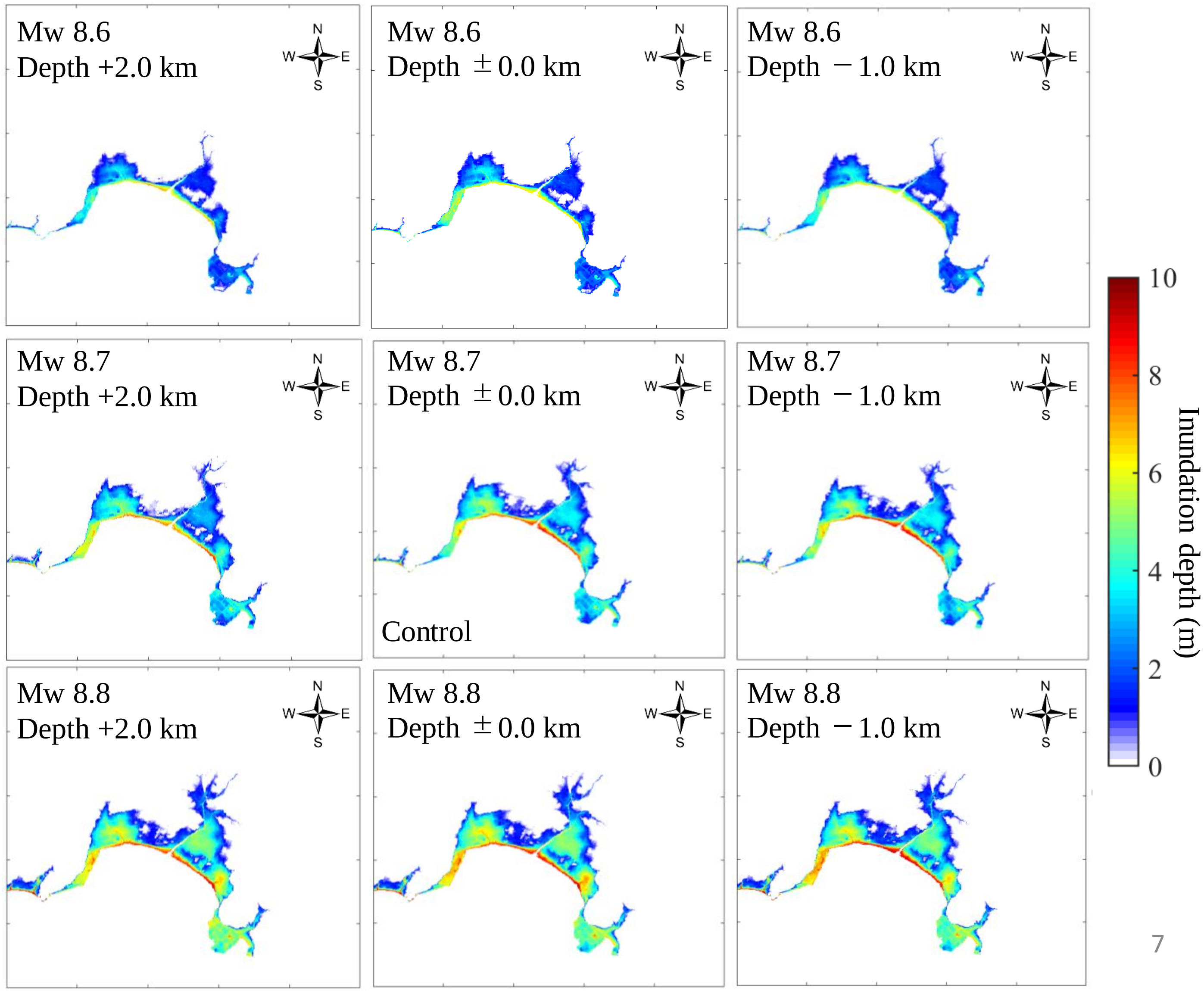
Calculations for all 27 cases

Uncertainty of earthquake fault

- Consideration of changes in fault slip (Mw 8.7±0.1) and fault depth (+2km, ±0km, -1km) → Tsunami numerical simulations for 3 x 3 = 9 cases

Center asperity model （Kamakura）



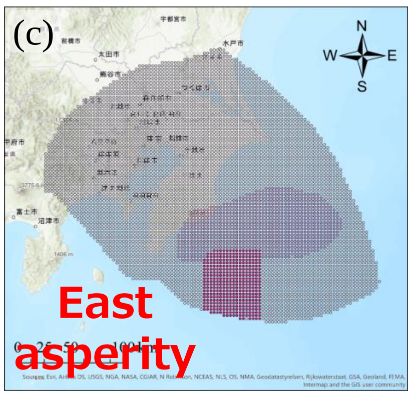


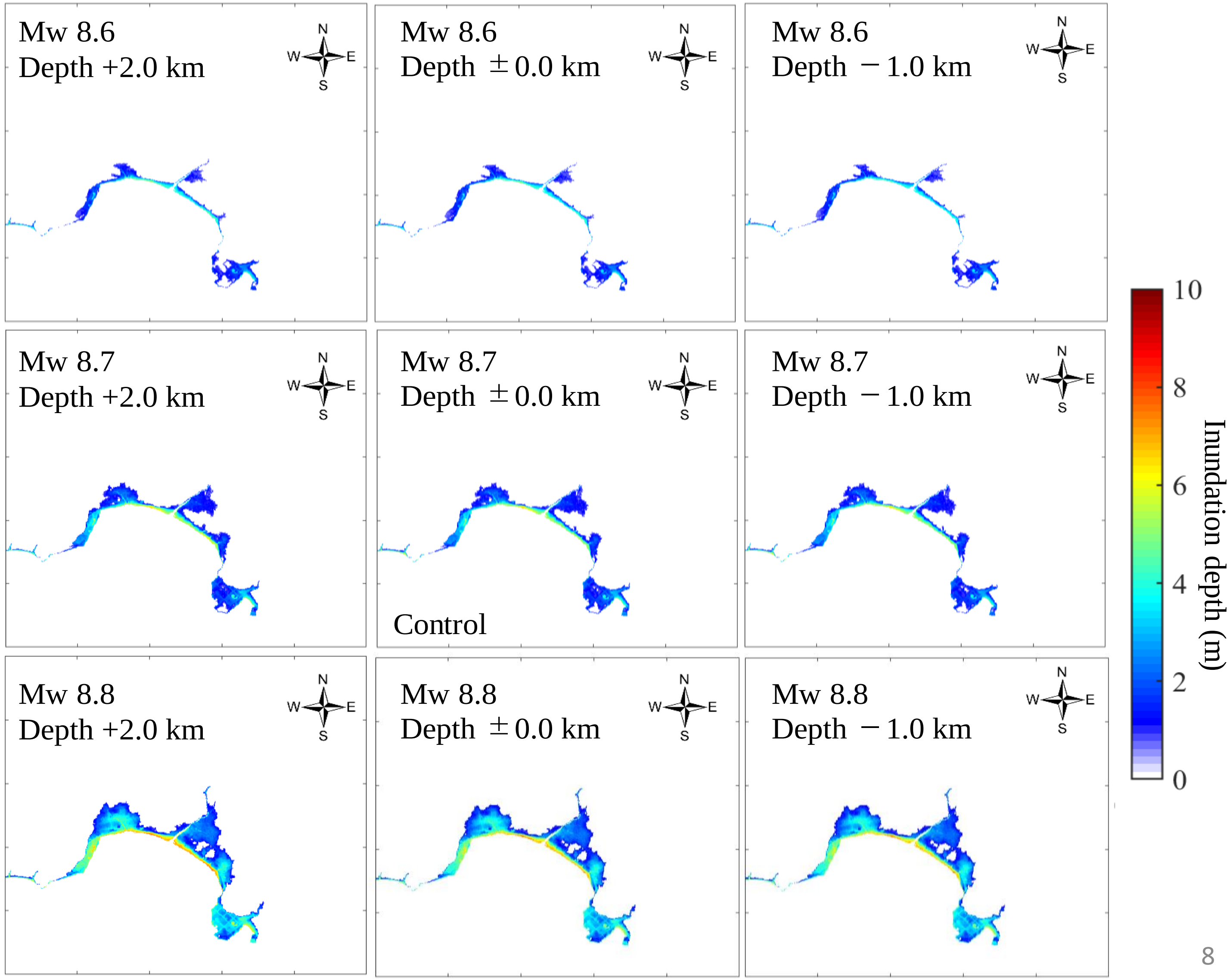
<u>**Calculations for all 27 cases**</u>

Uncertainty of earthquake fault

- Consideration of changes in fault slip (Mw 8.7±0.1) and fault depth (+2km, ±0km, -1km) → Tsunami numerical simulations for 3 x 3 = 9 cases

East asperity model (Kamakura)





Calculations for all 27 cases

Eigenmode of Inundation Depth Distribution (Singular Value Decomposition)

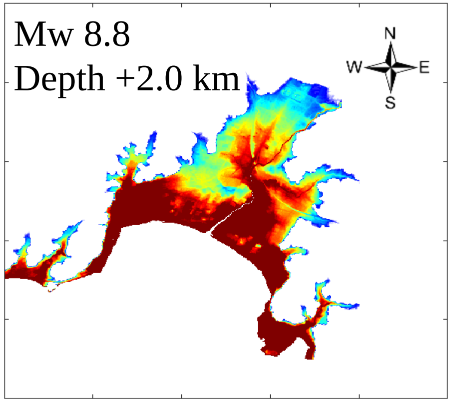
- We created a data matrix $\boldsymbol{X}$ for the inundation depth value (row direction) and the analytical case (column direction).

$$\boldsymbol{X} = \begin{bmatrix} x_{11} & \cdots & x_{1n} \\ & \ddots & \\ x_{m1} & \cdots & x_{mn} \end{bmatrix}$$

$\boldsymbol{m}$ dimensions (Spatial mesh numbers) = 301×401 =120701

$\boldsymbol{n}$ dimensions (Analysis case numbers) = 27

301



401

- We extracted the eigenmodes of the inundation depth by singular value decomposition of the data matrix $\boldsymbol{X}$.

$$\boldsymbol{X} = \boldsymbol{U\Sigma V^t}$$

Left singular vector $\boldsymbol{U}$

$$\boldsymbol{U} = \begin{bmatrix} | & & | \\ \boldsymbol{u_1} & \cdots & \boldsymbol{u_n} \\ | & & | \end{bmatrix}$$

Singular vector

$$\boldsymbol{\Sigma} = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \lambda_n \end{bmatrix}$$

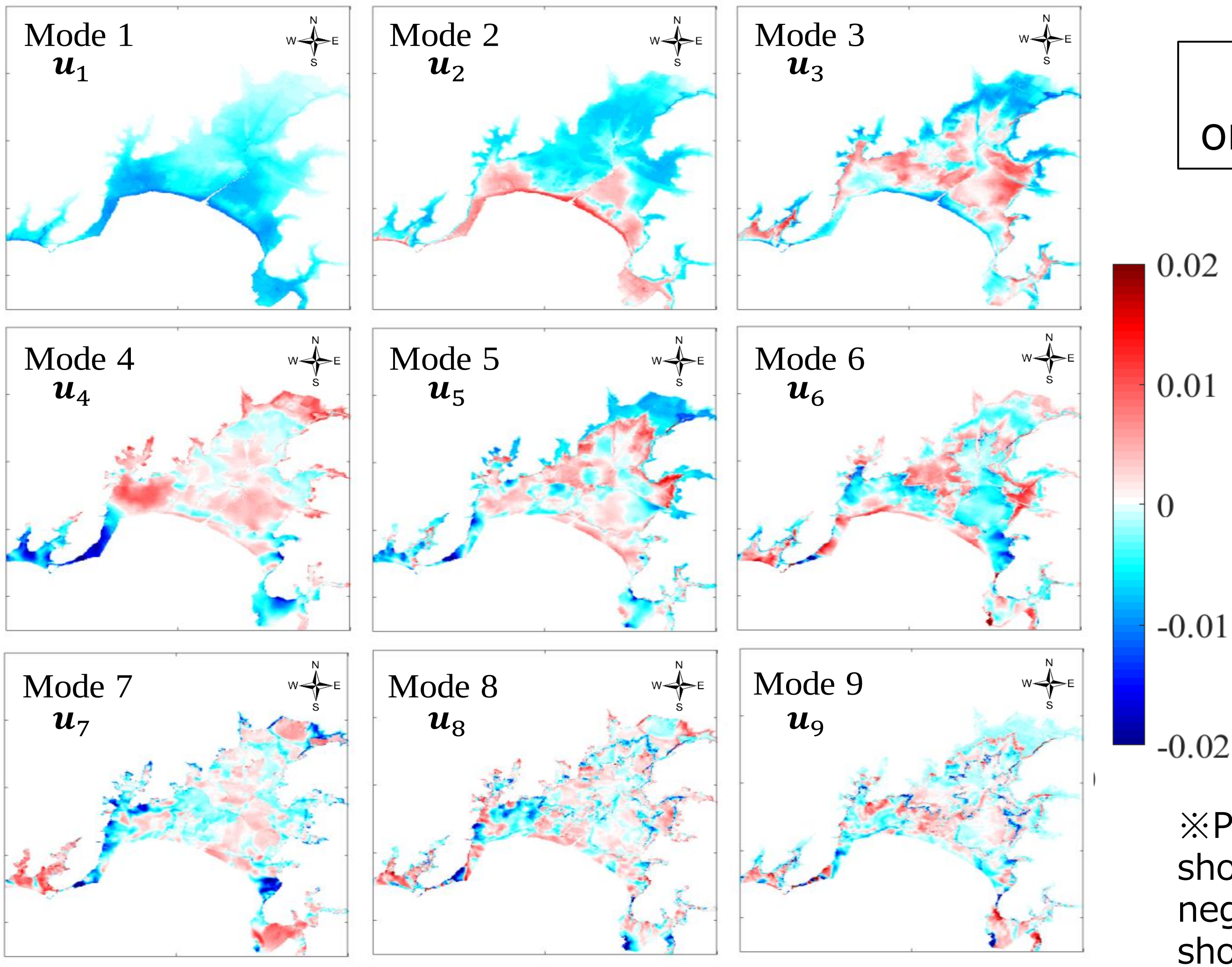
Right singular vector $\boldsymbol{V}$

$$\boldsymbol{V} = \begin{bmatrix} - & \boldsymbol{v_1} & - \\ & \vdots & \\ - & \boldsymbol{v_n} & - \end{bmatrix}$$

- We can understand the information about the spatial correlation between meshes of inundation data by the left singular vector $\boldsymbol{U}$

Extracted eigenmodes (Left singular vector $\boldsymbol{u}_k$)

- Positive correlation between meshes of the same sign and negative correlation between meshes of different signs are expressed.



Spatial mode only from 1 to 9

※Positive values are shown in red, and negative values are shown in blue.

Spatial distribution of $\boldsymbol{u}_j$(j = 1, ⋯, 9) constituting the left singular vector $\boldsymbol{U}$ (Spatial mode)

Construction of surrogate model

- The inundation depth data vector $\boldsymbol{x}_j$ for a given analysis case j can be expressed as follows

$$\boldsymbol{X} = \boldsymbol{U\Sigma V}^t$$

$$\boldsymbol{X} = \begin{pmatrix} | & \cdots & | \\ \boldsymbol{x}_1 & \cdots & \boldsymbol{x}_j \\ | & \cdots & | \end{pmatrix} = \begin{pmatrix} | & \cdots & | \\ \boldsymbol{u}_1 & \cdots & \boldsymbol{u}_j \\ | & \cdots & | \end{pmatrix} \begin{pmatrix} \boldsymbol{\lambda}_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \boldsymbol{\lambda}_j \end{pmatrix} \begin{pmatrix} - & \boldsymbol{v}_1 & - \\ \vdots & \vdots & \vdots \\ - & \boldsymbol{v}_j & - \end{pmatrix}^T$$

$$\boldsymbol{x}_j = \sum_{k=1}^{n} \boldsymbol{u}_k (\lambda_k v_{kj}^T) = \sum_{k=1}^{n} (\lambda_k v_{jk}) \boldsymbol{u}_k = \sum_{k=1}^{n} (\alpha_{jk}) \boldsymbol{u}_k$$

α_{jk} : Coefficient of jth case corresponding mode k

n :Number of analysis case（n=27）

- $\boldsymbol{x}_j$ can be expressed as a linear sum of the left singular vector $\boldsymbol{u}_k$ and the coefficients α_{jk}.
- Using this relationship, we can obtain the sample vector of $\boldsymbol{x}_j$ by giving the coefficients α_{jk} through a Monte Carlo simulation that takes into account the uncertainty of the fault parameters.

Estimation of coefficients α_{jk} corresponding to each mode (in case of 2 variables)

- We estimated the coefficients α_{jk} to the fault parameters by using Gaussian Process Regression (GPR).

Gaussian Kernel

$$\boldsymbol{\kappa}(\boldsymbol{x}_i, \boldsymbol{x}_j) = \exp\left(-\frac{(\boldsymbol{x}_i - \boldsymbol{x}_j)^T(\boldsymbol{x}_i - \boldsymbol{x}_j)}{\sigma^2}\right)$$

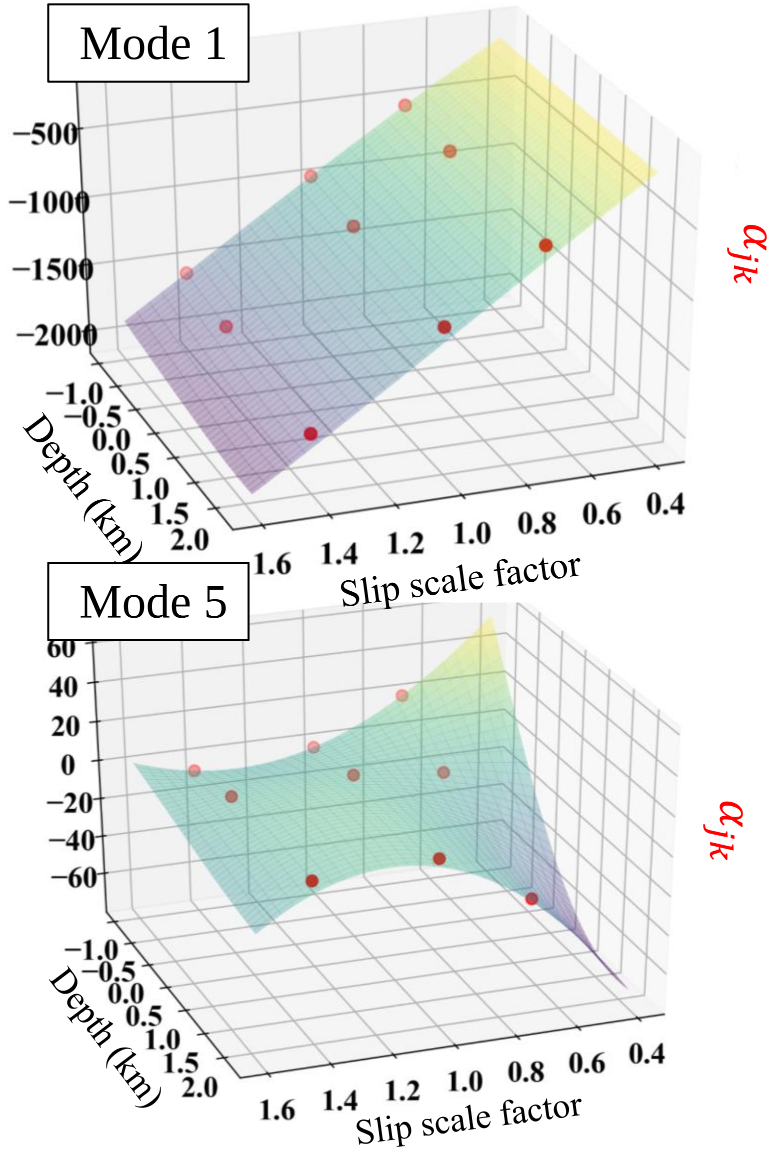
Joint prior distribution of $\mathbf{f}$ and $\mathbf{f}_*$

$$\begin{bmatrix}\mathbf{f}\\ \mathbf{f}_*\end{bmatrix} \sim N\left(0, \begin{bmatrix}\boldsymbol{\kappa}(X,X) & \boldsymbol{\kappa}(X,X_*)\\ \boldsymbol{\kappa}(X_*,X) & \boldsymbol{\kappa}(X_*,X_*)\end{bmatrix}\right)$$

Bayesian inference

Posterior predictive distribution $\mathbf{f}_*|\mathbf{f}$ that follows the prediction $\mathbf{f}_*$ given the training function $\mathbf{f}$

$$\begin{cases} \boldsymbol{m}_* = \boldsymbol{\kappa}(X,X_*)^T\boldsymbol{\kappa}(X,X)^{-1}\,\mathbf{f} \\ \boldsymbol{V}_* = \boldsymbol{\kappa}(X_*,X_*) - \boldsymbol{\kappa}(X,X_*)^T\boldsymbol{\kappa}(X,X)^{-1}\boldsymbol{\kappa}(X,X_*) \end{cases}$$



Estimation examples of coefficients α_{jk} (a) Mode 1, (b)Mode 5. The curved surface represents the mean value. The red dots are from the results of tsunami numerical calculation.

Probability distribution of slip scale, depth and asperity

Variables	Average	Standard dev.	Probability distribution	Basis for variability setting
Slip scale factor ***S***	1.00	0.35	Log normal	Mw is suggested to vary by about ±0.1, considering previous earthquakes with the same fault area. (The Nuclear Civil Engineering Committee, JSCE, 2011)
Fault depth ***D*** [km]	0.00	2.0	Normal	There was a standard deviation of several kilometers based on the results of research evaluating the depth of the upper edge of the Philippine Sea Plate.
Asperity location ***A***	Uniform distribution [-1.0~1.0]			West asperity : -1 Center asperity : 0 East asperity : 1

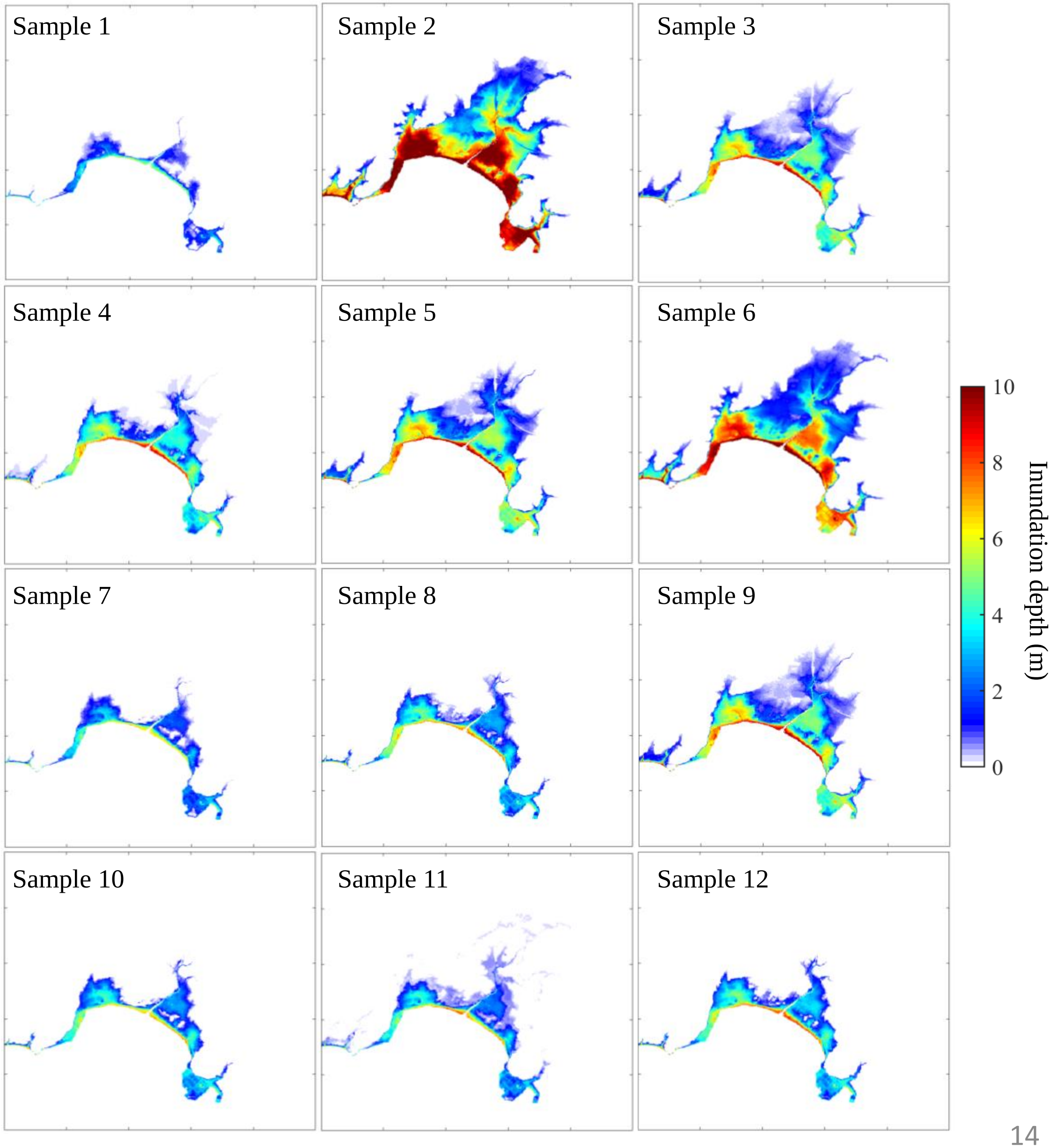
- Finally, we can reconstruct the inundation depths and distributions by setting the probability distribution of the fault parameters.

Tsunami inundation distribution samples from the surrogate model

- We estimated the coefficients α_{jk} using the probability distributions of the input parameters, and generated a number of samples of tsunami inundation depth distributions considering spatial correlation from the linear sum of each mode.

- The samples generated by the Monte-Carlo simulation show a good representation of the original tsunami inundation depth distribution.

Monte-Carlo samples of tsunami inundation distributions



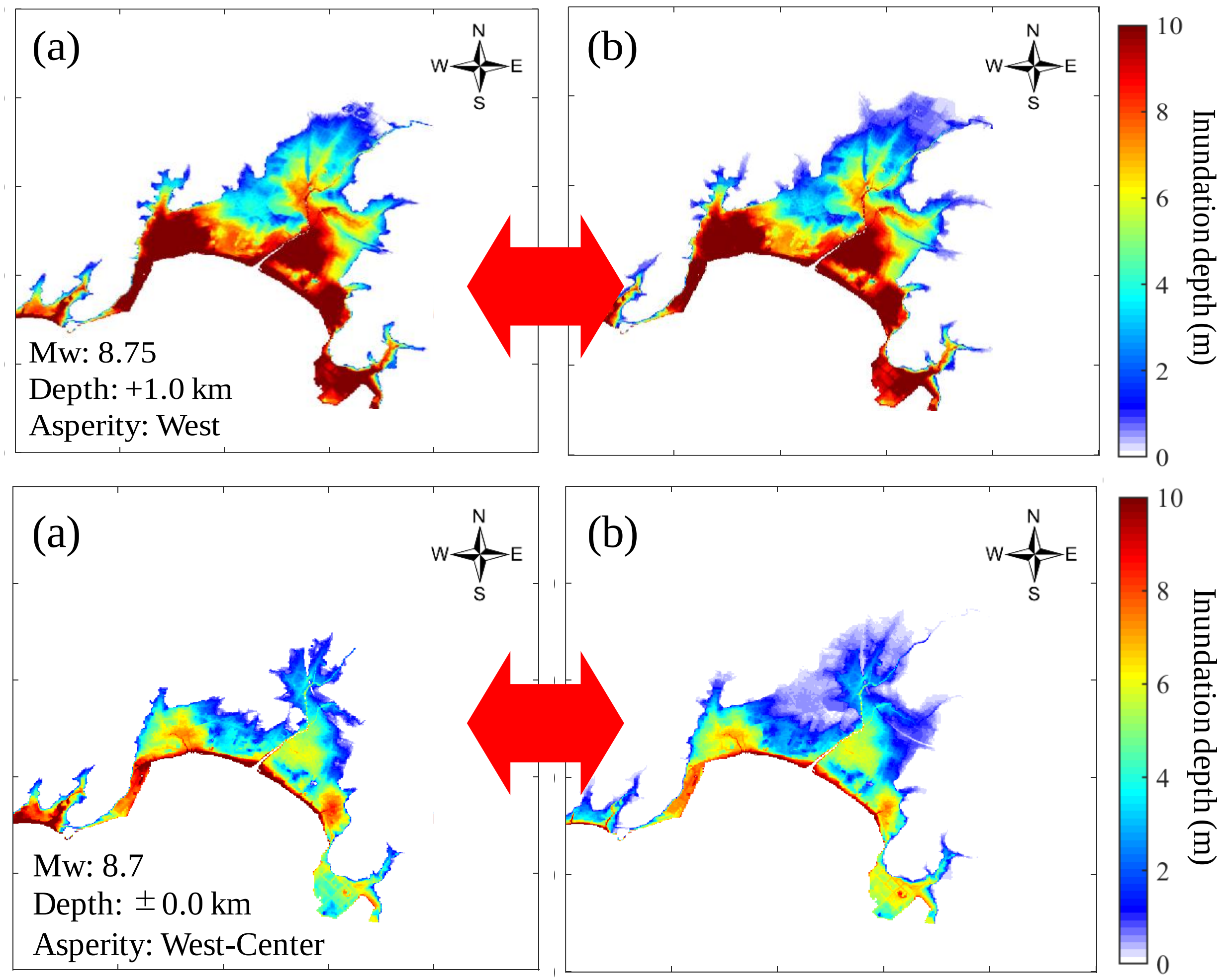
Verification of the surrogate model

- Comparison of the results of tsunami numerical simulation and surrogate model for different settings than the first 27 cases.

- **<u>We found good agreement with the results of the surrogate model and tsunami numerical simulations.</u>**

Tsunami numerical simulation

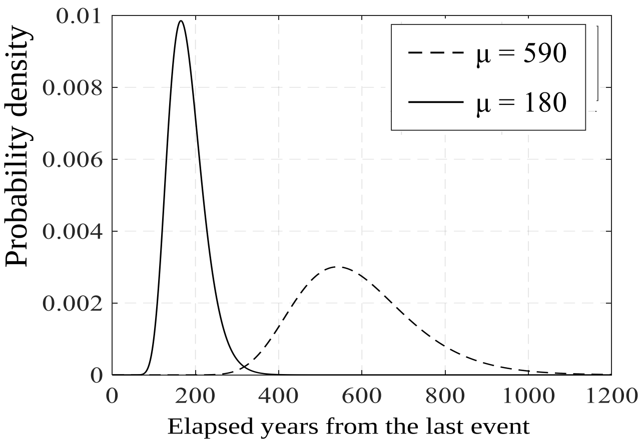
Surrogate model



Probabilistic tsunami inundation assessment

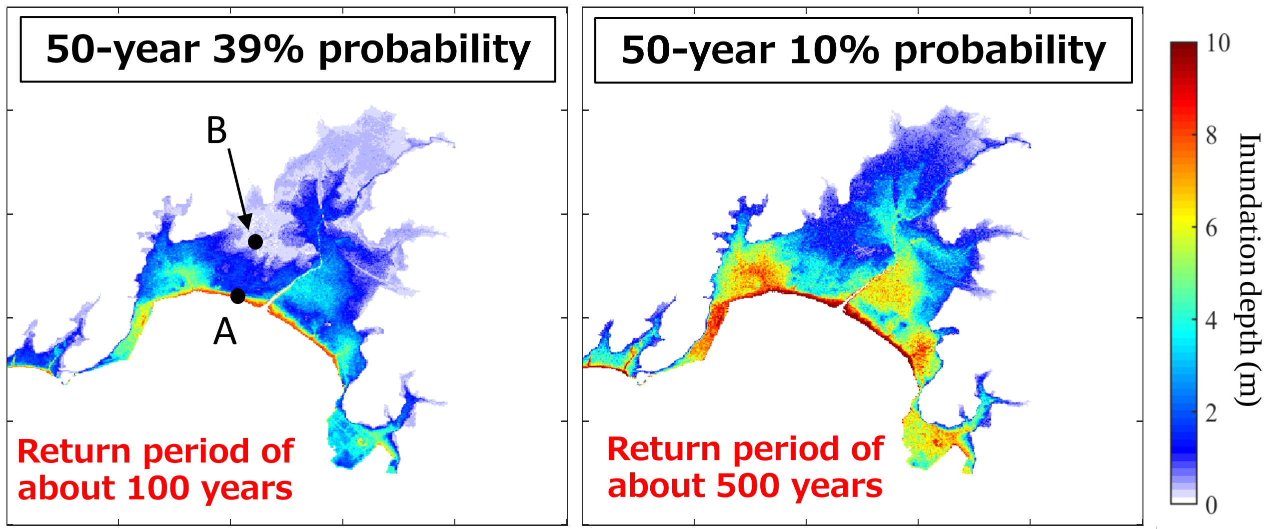
- We performed a probabilistic tsunami hazard assessment by incorporating a time-dependent probability model (BPT distribution) of the Sagami Trough megathrust earthquake using 10,000 Monte Carlo simulations.

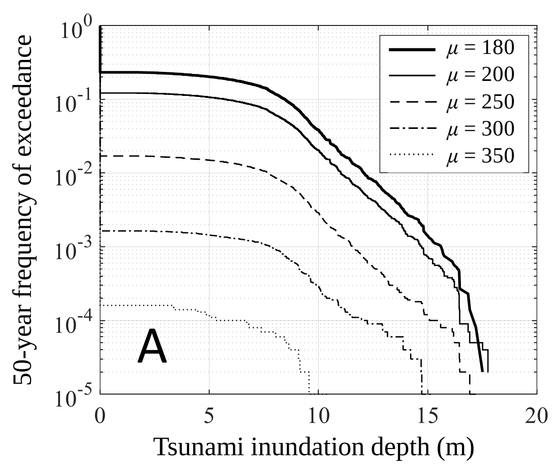
Brownian Passage Time (BPT) Distribution

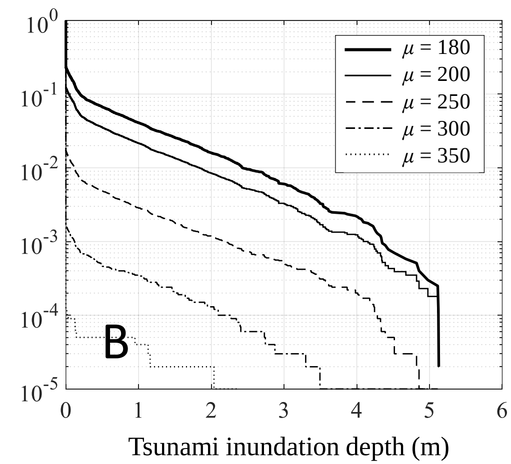


$$P(t) = \sqrt{\frac{\mu}{2\pi\alpha^2 t^3}}\, exp\left(-\frac{(t-\mu)^2}{2\mu\alpha^2 t}\right)$$

- t : time lapse from last earthquake
- μ : average year of earthquake occurrence interval
- α : variability of earthquake occurrence interval





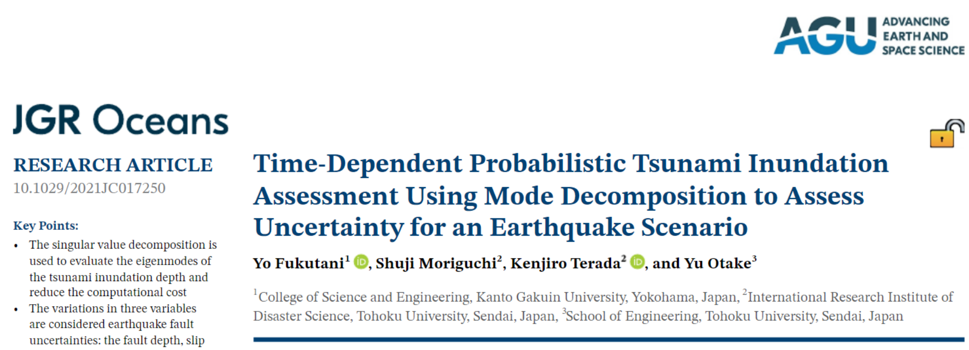


Summary

- We generated random distributions of the tsunami inundation depth **by applying SVD to a limited number of tsunami inundation depths and evaluating the spatial correlation of those inundation depths**, and we proposed a method **to evaluate tsunami hazard curves and probabilistic tsunami inundation depths** within a certain area considering the imminent occurrence of an earthquake.
- Based on the good agreement between the samples generated by the proposed surrogate model and the nonlinear long wave, we can conclude that the proposed probabilistic tsunami inundation assessment using the proposed method is sufficiently accurate and practical.

【Future work】

- We have a plan to use fault models generated from a physical stochastic slip model and fault models with varying fault parameters, while in this study, the slip distribution of faults was generated simply in one dimension.



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Thank you for your attention.